1 Problem set 1, Due October 31

1. Read and take notes on Chatterjee's $SO(N)$ paper. Write down honestly what parts you did and did not understand.

2. Read and take notes on either the Eynard paper or the Guionnet paper posted on the course website (your choice). Be aware that the notation may be a bit different from what we did in class (extra factors of N in matrix ensemble definitions, dual map instead of map, etc.)

3. Let G be a finite connected subset of $\mathbb{Z}^2$ and let h be an instance of the discrete Gaussian free field on $\mathbb{Z}^2$ (as defined modulo additive constant; see *Gaussian free fields for mathematicians*). Then the discrete Laplacian of h is independent of the additive constant. Compute $\left\langle \prod_{v \in G} (-\Delta h)^k \right\rangle$ in combinatorial terms.

4. Consider a Ginibre $N \times N$ matrix A, where the entries are i.i.d. complex Gaussian with mean zero, normalized so that if X is an entry of the matrix then $E[X\overline{X}] = 1$. Give a combinatorial interpretation of the quantity $\left\langle (\mathrm{Tr} AA^t)^n \right\rangle$. (Here A^t denotes conjugate transpose of A. Try the case $N = 1$ first.)

5. How about $\left\langle (\mathrm{Tr} AA^tAA^t)^n \right\rangle$?

6. Write down the joint law for the set of eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of AA^t? (Look up Lalley's notes: *The Gaussian and Wishart Ensembles: eigenvalue densities* and consider special case when $p = n$. Is this Corollary 7?) Explain what happens to this law when we replace the Ginibre ensemble law with the same law *weighted* by $\left[(\mathrm{Tr}(AA^t))^2 - \mathrm{Tr} AA^tAA^t\right]^k = \left((\sum \lambda_i)^2 - \sum \lambda_i^2\right)^k$. (Try first giving the law of $\sum \lambda_i$, then the law of the set λ_i conditioned on the sum.) What can be said about (suitably normalized) large k limits? Does the law of A (rescaled by an appropriate constant) start to look more like the law of a unitary matrix chosen from some Haar measure?

7. Suppose we have an independent Ginibre matrix for each directed edge (edge reversal corresponds to taking conjugate tranpose) of a lattice graph obtained by restricting $\mathbb{Z}^d$ to an $n \times n$ box. Compute $E \prod_p (W_p^k)$ combinatorially where p ranges over oriented plaquettes. Compute $\left\langle W_{\ell_1} W_{\ell_2} \cdots W_{\ell_n} \right\rangle$ combinatorially for a general collection of oriented

loops (which may include some number of copies of each oriented plaquette).

8. Look up eigenvalue distribution formulas for GUE, GOE, CUE, and Ginibre ensembles and write them down with a sentence or two about what makes each one interesting.

9. Explain the matrix tree theorem and determinant Laplacian calculation for a simple graph: let's say the triangle.

10. Come talk to me at some point about open problems and/or your final project.

2 Problem set 2, Due November 14

1. Just to make sure you have the definitions in mind: Let B_t be standard Brownian motion and $f(x) = x^3 + e^x$. Compute $df(B_t)$ using the formalism of Ito's formula.

2. Just to make sure you have the definitions in mind, work out an explicit example (as small as you like) of the Cori-Vaquelin-Schaeffer, the Mullin bijection, the hamburger cheeseburger bijection, and the bipolar planar map bijection.

3. Just to make sure you have the definitions in mind, generate a uniform random spanning tree on a small graph (by hand) using coin tosses and Wilson's algorithm.

4. Just to make sure you have the definitions in mind: if you have the range of a stable subordinator of parameter α and the range of another stable subordinator of parameter β, what conditions on β and α ensure the the intersection of these two sets is almost surely empty? Use the Bessel process relationship to explain your answer.

5. Read and take notes on Berestycki's *Introduction to the Gaussian free field and Liouville Quantum Gravity.* (You may also consult *Gaussian free fields for mathematicians* and the introduction to *Liouville quantum gravity and KPZ.*) Try working through some of the exercises.

6. Come talk to me at some point about open problems and/or your final project.

3 Problem set 3, due November 28

1. Read and take notes on *Schramm-Loewner evolution* by Berestycki and Norris. (You may also consult the notes by Lawler, by Kager/Neinhuis, and by Werner on the course web page.) Try working out a few of the things left as exercises.

2. Read and take notes on *one* of the references from the *Selected references on universal object relationships* portion of the webpage (your choice).

3. Read and take notes on one of the papers about growth models (your choice).

4. Come talk to me at some point about open problems and/or your final project.

4 Final Project, due last day of class, Dec. 12

Also, talk to me some time befor edue date about open problems and/or the final project.